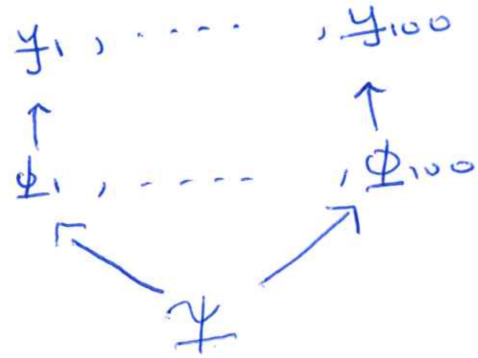
y_{ij} = the score of student i at school j

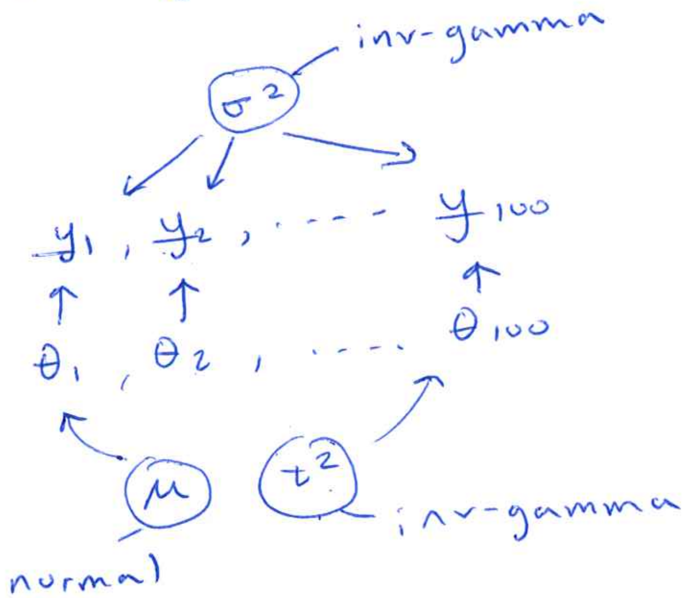
$\underset{\sim}{y}_j$: vector of scores at school j

n_j : the # of students at school j.

Data :



Model : $y_{ij} \mid \theta_j, \sigma^2 \sim N(\theta_j, \sigma^2)$



$\theta_j \mid \mu, \tau^2 \overset{iid}{\sim} N(\mu, \tau^2)$

To implement a Gibbs sampler, we require the full conditional posterior for each unknown:

$p(\theta_j \mid \cdot) \quad \forall j$

$p(\sigma^2 \mid \cdot)$

$p(\tau^2 \mid \cdot)$

$p(\mu \mid \cdot)$

* Recall: Each full cond'l posterior is proportional to joint posterior:

$$p(\theta_1, \dots, \theta_m, \sigma^2, \tau^2, \mu \mid y_1, \dots, y_m) \propto p(y_1, \dots, y_m \mid \theta_1, \dots, \theta_m, \sigma^2) \cdot p(\theta_1, \dots, \theta_m \mid \mu, \tau^2) \cdot p(\sigma^2) p(\tau^2) p(\mu)$$

Posterior Predictive Prob:

$$p(\underbrace{Y^*_{i,51}}_{\text{school 51}} > \underbrace{Y^*_{i,41}}_{\text{school 41}} \mid \underbrace{y_1, \dots, y_m}_{\text{data}})$$

$$= \int p(Y^*_{i,51} > Y^*_{i,41} \mid \theta_{41}, \theta_{51}, \sigma^2)\, p(\theta_{41}, \theta_{51}, \sigma^2 \mid y_1, \dots, y_m)\, d\theta_{41}\, d\theta_{51}\, d\sigma^2$$